Unofficial Solutions of “Solar Energy” Mock exam (WT2021/22)

This is a rewrite of the sample solution (that is not handed out to the students) taken from a hybrid tutorial.

Task 1: Solar spectrum

a. Curve i) $\longrightarrow$ AM0

   Curve ii) $\longrightarrow$ AM1,5G, 1,5 atmosphere thickness, corresponds to a solar zenith angle of $\xi = 48{,}2°$. The “G” stands for global and includes incoming scattered light from the atmosphere at various angles (slight blue shift of the spectrum).

b. The spectrum outside the atmosphere is referred to as “AM0”, meaning “zero atmospheres”. The difference in comparison with the perfect black body arises from the Fraunhofer emission lines and the absorption lines if the atoms in the sun. Additionally, colder regions in the photosphere are leading to absorption. Difference to AM0 due to: ozone, aerosol, water vapour and oxygen absorption and Rayleigh scattering in the atmosphere.

c. Given: $P_{sun} = 2{,}4 \cdot 10^{45}\,\frac{\text{eV}}{\text{s}}$, $\quad T_{sun} = 5778\,\text{K}$, $\quad d_{sun} = 1{,}391 \cdot 10^{6}\,\text{km}$

$$P_{sun} = \sigma \cdot A_{sun} \cdot {T_{sun}}^4 = \sigma \cdot 4\pi \cdot \left(\frac{d_{sun}}{2}\right)^2 \cdot {T_{sun}}^4 = \sigma \cdot \pi \cdot d_{sun} \cdot {T_{sun}}^4$$

$$\Leftrightarrow \sigma = \frac{P_{sun}}{\pi \cdot {d_{sun}}^2 \cdot {T_{sun}}^4} = \frac{2{,}4 \cdot 10^{45}\,\text{eV} \cdot 1{,}602 \cdot 10^{-19}\,\frac{\text{W}}{\text{eV}}}{\pi \cdot \left(1{,}391 \cdot 10^{9}\,\text{m}\right)^2 \cdot \left(5778\,\text{K}\right)^4} = 5{,}67 \cdot 10^{-8}\,\frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}$$

d. Large bandgap

   - Advantage: Low thermalisation losses, high extraction energy of electrons meaning we get a high V_{OC}.
   - Disadvantage: High transparency losses, fewer absorbed photons lead to a lower I_{SC}.

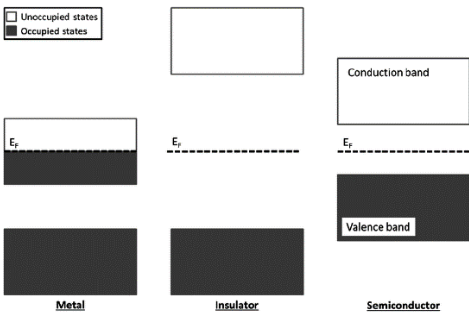
   Low bandgap

   - Disadvantage: High thermalisation losses, low extraction energy of electrons meaning we get a low V_{OC}.
   - Advantage: Low transparency losses, more absorbed photons lead to a higher I_{SC}.

e. Up-conversion, down-conversion, hot electron capture, tandem solar cells, solar concentrators.

Task 2: Physics of Semiconductor

a. See picture



Source: Optical design guidelines for spectral splitting photovoltaic systems : a sensitivity analysis approach - Scientific Figure on ResearchGate. Available from: https://www.researchgate.net/figure/Band-structure-of-solids-Band-structure-of-metals-insulators-and-semiconductors-are_fig1_279810779 [accessed 16 Feb, 2022]

b. C-Si has an indirect bandgap, while GaAs has a direct bandgap. Much higher and steeper absorption coefficient for direct bandgap.

c. Diffusion (Fick's law): $\underbrace{j_1}_{\substack{\text{diffusion}\\\text{current}\\\text{density}}} = - \underbrace{q}_{\substack{\text{elementary}\\\text{charge}}} \cdot \underbrace{D}_{\substack{\text{diffusion}\\\text{coefficient}}} \cdot \underbrace{\frac{\partial n}{\partial x}}_{\substack{\text{electron}\\\text{concentration}\\\text{gradient}}}$, dominant in crystalline silicon.

Drift (Ohm's law): $\underbrace{j_2}_{\substack{\text{drift}\\\text{current}\\\text{density}}} = \underbrace{q}_{\substack{\text{elementary}\\\text{charge}}} \cdot \underbrace{n}_{\substack{\text{charge carrier}\\\text{density}}} \underbrace{\mu}_{\text{mobility}} \cdot \underbrace{E}_{\substack{\text{electric}\\\text{field}}}$, dominant in amorphous silicon.

The crystalline silicon has an ordered structure. In the amorphous silicon, the electrons are disordered which leads to an electric field.

d. Given: $\lambda = 620\,\text{nm}$, 80% absorption

$$E_G = \frac{h \cdot c}{\lambda \cdot q} = \frac{6{,}626 \cdot 10^{-34}\,\text{Js} \cdot 2{,}998 \cdot 10^{8}\,\frac{\text{m}}{\text{s}}}{620 \cdot 10^{-9}\,\text{m} \cdot 1{,}602 \cdot 10^{-19}\,\text{C}} = 2\,\text{eV}$$

By reading the graph, we get: $\alpha_{\text{c-Si}} = 3 \cdot 10^{3}\,\frac{1}{\text{cm}}$ and $\alpha_{\text{a-Si}} = 2 \cdot 10^{4}\,\frac{1}{\text{cm}}$

Using Lambert-Beer-Law:

$$I = I_0 \cdot e^{(-\alpha \cdot d)} \Leftrightarrow \frac{I}{I_0} = e^{(-\alpha \cdot d)} \Leftrightarrow d = -\frac{\ln\left(\frac{I}{I_0}\right)}{\alpha}$$

With $\frac{I}{I_0} = 0{,}2$ we get for crystalline silicon:

$$d_{\text{c-Si}} = -\frac{\ln(0{,}2)}{3 \cdot 10^{3}\,\frac{1}{\text{cm}}} = 5{,}36\,\mu\text{m}$$

and for amorphous silicon:

$$d_{\text{a-Si}} = -\frac{\ln(0{,}2)}{2 \cdot 10^{4}\,\frac{1}{\text{cm}}} = 805\,\text{nm}$$

Task 3: Solar cell

a. 1: top-contact
   2: glass / encapsulation
   3: transparent conductive oxide (TCO) / charge selective layer
   4: absorber layer
   5: transparent conductive oxide (TCO) / charge selective layer
   6: Back contact / metallization

b. Given: $X = 10$, $V_{\text{OC}_0} = 650\,\text{mV}$, $I_{\text{SC}_0} = 30\,\text{mA}$, $n_{\text{ideality}} = 1{,}0$, $T = 25\,°\text{C}$, $FF = 0{,}7$, $P_{\text{sun}} = 0{,}5\,\text{W}$

By concentration we get $I_{\text{SC}} = X \cdot I_{\text{SC}_0} = 10 \cdot 30\,\text{mA} = 300\,\text{mA}$.

Variation of the open-circuit voltage:

$$V_{\text{OC}} = \frac{n_{\text{ideality}} \cdot k_B \cdot T}{q} \cdot \ln\left(\frac{I_{\text{SC}}}{I_0}\right) = \frac{n_{\text{ideality}} \cdot k_B \cdot T}{q} \cdot \ln\left(\frac{X \cdot I_{\text{SC}_0}}{I_0}\right)$$
$$= \underbrace{\frac{n_{\text{ideality}} \cdot k_B \cdot T}{q} \cdot \ln\left(\frac{I_{\text{SC}_0}}{I_0}\right)}_{=V_{\text{OC}_0}} + \frac{n_{\text{ideality}} \cdot k_B \cdot T}{q} \cdot \ln(X)$$

$$= V_{OC_0} + \frac{1{,}0 \cdot 1{,}38 \cdot 10^{-23}\,\frac{\mathrm{m^2 \cdot kg}}{\mathrm{s^2 \cdot K}} \cdot 298\,\mathrm{K}}{1{,}602 \cdot 10^{-19}\,\mathrm{C}} \cdot \ln(10) = 650\,\mathrm{mV} + 60\,\mathrm{mV} = 710\,\mathrm{mV}$$

Efficiency:

$$\eta = \frac{P_{\mathrm{MPP}}}{P_{\mathrm{sun}}} = \frac{V_{\mathrm{OC}} \cdot I_{\mathrm{SC}} \cdot FF}{P_{\mathrm{sun}}} = \frac{710\,\mathrm{mV} \cdot 300\,\mathrm{mA} \cdot 0{,}7}{0{,}5\,\mathrm{W}} = \frac{0{,}71\,\mathrm{V} \cdot 0{,}3\,\mathrm{A} \cdot 0{,}7}{0{,}5\,\mathrm{W}} = 0{,}298 \mathrel{\hat{=}} 29{,}8\,\%$$

c. Decrease in V_{OC}:

$$V_{\mathrm{OC}}{}' = V_{\mathrm{OC}} - \frac{\partial V_{\mathrm{OC}}}{\partial T} \cdot \Delta T = 710\,\mathrm{mV} - 2\,\frac{\mathrm{mV}}{\mathrm{K}} \cdot 50\,\mathrm{K} = 610\,\mathrm{mV}$$

d. In concentrated PV, particular attention should be posed on temperature effects. Cooling is essential to exploit the full potential of concentrated PV.

Task 4: Advanced optics and energy yield modelling

a. To minimize reflection using a thin film ⟶ geometric mean of refractive indices:

$$n_1 = \sqrt{n_0 \cdot n_2} = \sqrt{1 \cdot 4} = 2$$

Using quarter-wave anti reflection coating:

$$d_1 = \frac{\frac{\lambda}{4}}{n_1} = \frac{\frac{530\,\mathrm{nm}}{4}}{2} = 66{,}25\,\mathrm{nm}$$

b. Which physical effect is responsible for losses.

c. 1: mostly strong absorption in the top layers

2: parasitic absorption & reflection losses.

3: reflection and transmission losses increased due to decreased absorption for photons with energy close to bandgap

Optimize optics to reduce reflection and parasitic losses, Anti-reflective coating (ARC), layer thickness, textures, refractive index gradually increasing